We are looking to find a quadratic function that satisfies

$$f(0) = 0,\ f(50) = 181, \text{and}\ f(116) = 727, \tag{1}$$

where we estimated based on the graph of the cross-sectional area of the dam that $f(50)$ should be around 181.

A general quadratic function has the form: $f(x) = a \cdot x^2 + b \cdot x + c$, for some given real numbers a, b and c. Thus, we need values for a, b and c such that the function passes through the three given points stated in (1).

Note that by plugging 0 into our general formula for $f(x)$ we have

$$f(0) = a \cdot 0^2 + b \cdot 0 + c \text{ and } f(0) = 0 \text{ from (1).}$$

So,

$$a \cdot 0^2 + b \cdot 0 + c = 0$$

i.e. $c = 0$.

Also,

$$f(50) = a \cdot 50^2 + b \cdot 50 + c \text{ and } f(50) = 181.$$

Since $c = 0$ we get

$$2500a + 50b = 181\,. \tag{2}$$

Also,

$$f(116) = a \cdot 116^2 + b \cdot 116 + c \text{ and } f(116) = 727.$$

Since $c = 0$ we get

$$116^2 a + 116b = 727\,. \tag{3}$$

Equations (2) and (3) form a system of equations of two unknowns a and b.

Let's solve our system

$$\begin{aligned} 2500a + 50b &= 181\,, \\ 116^2 a + 116b &= 727\,. \end{aligned}$$

We will use substitution. First equation gives

$$\begin{aligned} 2500a &= 181 - 50b\,, \\ a &= \frac{181 - 50b}{2500}\,. \end{aligned}$$

Next we will substitute a into the second equation of our system to obtain

$$116^2 \cdot \left(\frac{181 - 50b}{2500}\right) + 116b = 727\,, \tag{4}$$

i.e. (multiply by 2500 the entire equation and move the constant term from left side to right side)

$$\begin{aligned} -672,800b + 290,000b &= -618,036\,, \\ -382,800b &= -618,036\,, \\ b &= 1.6145\,. \end{aligned}$$

Now, lets use the value of b to find a,

$$a = \frac{181 - 50b}{2500} = \frac{181 - 50 \cdot 1.6145}{2500} = 0.04011\,.$$

Thus our function equals to

$$f(x) = a \cdot x^2 + b \cdot x + c = 0.04011x^2 + 1.6145x\,,$$

since we obtained $a = 0.04011$, $b = 1.6145$ and $c = 0$.

Check that:

$$\begin{aligned} f(0) &\simeq 0 \\ f(50) &\simeq 181 \\ f(116) &\simeq 727 \end{aligned}$$

Note: You may not get exact values due to the rounding error.